Comment on "Two-Equation Model Consistent with Second Law"

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THE authors are to be congratulated for providing some light on the significance of the Second Law of Thermodynamics on two-equation turbulence models. The statements made by the Second Law on irreversibility are especially pertinent to turbulence and may in the future allow us a better understanding of the turbulent energy cascade and transition processes.

The purpose of this Comment is to point out a few inconsistencies in the turbulence model presented in the above paper, which may help save practitioners of turbulence modeling from pointless programming. First, it is clear with reference to an earlier paper by Busnaina et al.[2] that the value of C^μ used was the usual 0.09, not 0.99. The lack of a low-Reynolds-number damping expression for the turbulent viscosity indicates that the model is not suitable for near-wall or other low-Reynolds-number applications.

A scrutiny of the proposed dissipation equation indicates that for the equation to balance in the near-wall region, $C^{\epsilon 3}$ must equal *half* of $C^{\epsilon 2}$ to insure a quadratic near-wall profile for turbulent kinetic energy, whereas the proposed constants' values were $C^{\epsilon 3} = C^{\epsilon 2} = 1.92$. In addition, since the authors were solving for dissipation itself rather than a dissipation variable, the presence of the turbulence kinetic energy k in the denominator of the two leading order terms

$$\left[4C^{\epsilon 3}\frac{\epsilon}{k}\left(\mu + \frac{\mu_t}{\sigma_k}\right)\left(\frac{\partial\sqrt{k}}{\partial x_i}\right)^2\right] \text{ and } \left(-\rho C^{\epsilon 2}\frac{\epsilon^2}{k}\right)$$

insured that a subtraction of two (nonequal) near-infinite quantities is required near the wall. The analysis of the behavior of the model is not helped by the fact that no boundary conditions are stated by Busnaina et al. in Refs. 1 or 2, nor by the coauthor Ahmadi in Ref. 3. One further criticism of the proposed model for the dissipation rate is that since the particular form of the diffusion term employed should vanish in the log-law regions of boundary layers (where dissipation is inversely proportional to wall distance), it should be the case that $C^{\epsilon 1} = C^{\epsilon 2}$ for production to balance dissipation. This may explain the authors' use of a very high value of $C^{\epsilon 1}$ of 1.69 rather than the consensus value of around 1.44 (Patel et al.[4]). The reason why the authors managed to get any results at all for the flow over a back step in spite of the difficulties outlined above is probably due to the coarse nature of their near-wall grid.[2] The model as it stands is almost certainly inherently grid-dependent.

Part of the reason for the problems in Busnaina et al.'s model may lie in the fact that the Second Law merely expresses an inequality, namely that of the change in entropy. To translate that inequality into an *equality* in order to solve for the dissipation involves the judicious use of closure assumptions (in Busnaina et al.'s case, the principal one being that for the free energy of turbulence), which may not be universally valid. It therefore behooves turbulence afficionados and casual users alike to be wary of replacing current best practice with unsupported hypotheses.

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Reply by Authors to F. Tarada

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WE would like to thank Dr. Tarada for his interest and for his comments on our recent paper.[1] In particular, his remarks concerning the significance of the Second Law in future research in turbulence modeling are well to the point.

As noted by Dr. Tarada and also in Ref. 2, $C^\mu = 0.09$ is the proper value of the constant. Furthermore, the model described in Ref. 1 clearly does not cover the low-Reynolds-number region nor was it claimed as such. Therefore, no damping expression for turbulent viscosity was needed.

Our thermodynamical argument shows that $C^{\epsilon 3} \leq C^{\epsilon 2}$. Dr. Tarada's suggestion that $C^{\epsilon 3} = C^{\epsilon 2}/2$ is an interesting possibility. However, our analysis showed that the results are not sensitive to the value used for $C^{\epsilon 3}$.

The additional terms that have appeared in the dissipation equation of the new model resemble those suggested for near wall flow regions.[3] However, our intention in Ref. 1 was not to describe near-wall flows, and a standard logarithmic law of the wall was used.

In Ref. 1 it was explicitly stated that a recent version of the TEACH computer code (STARPIC) was modified and used. It is well known that this code uses the standard law of the wall boundary conditions. Therefore, there was no need to emphasize the boundary conditions further.

For matching with the inertial sublayer, we have noticed that, for a constant σ^ϵ, the model requires that the values of $C^{\epsilon 1}$, $C^{\epsilon 2}$ must be equal. However, this improper choice of constants can be avoided by consideration of a mild dependence of σ^ϵ on turbulent Reynolds number. This would lead to proper recovery of the inertial sublayer flow with $C^{\epsilon 1}$ and $C^{\epsilon 2}$ being different. These additional details were omitted from Ref. 1 due to space limitation.

Although we did not cover the wall region, the grid used was comparable with those commonly used in the literature. The presence of numerical diffusion within the limits of the TEACH code had to be tolerated. However, we do not believe that the model is more grid-dependent than the standard k-ϵ model.

It is a fact that in the formulation of the model closure hypotheses consistent with the Second Law were needed. Some of these closure assumptions also could be improved in the future. Nevertheless, we believe that the Second Law is a sig-

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nificant law of nature and its implications in turbulence modeling need to be further explored.

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Comment on "Stresses and Rate of Twist in Single-Cell Thin-Walled Beams with Anisotropic Walls"

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BASED upon thin-walled beam assumptions, the author successfully obtained the interactive correlations between deformations of stretching, twisting, and bending, and applied loads of axial tension, torque, and bending moments for beams of anisotropic materials. Although, the conclusions were correct, the author appeared to be unaware of the earlier publications of Reissner and Tsai.[2] and Tsai[3] In these publications, a general class of anisotropic material was considered by using thin cylindrical shell theory. Closed-form solutions were obtained and effects of cross-sectional contraction were discussed. Limitation of material properties in a particular form and the assumption of neglecting the normal stress resultant N_s along the circumferential coordinate, as what were considered in the paper, were not required.

In discussing anisotropic thin-walled beams, the writer would like to share with the readers an important property that does not happen in isotropic beams nor can be determined by using the classic beam assumptions. Specifically, it is known that the stretching and bending rigidities of a beam remain unchanged between a closed cross section and the same cross section with a longitudinal slit if the material is isotropic. However, the rigidities may be significantly altered between the above-mentioned cross sections if the material is anisotropic. For orthotropically laminated thin-walled beams, the

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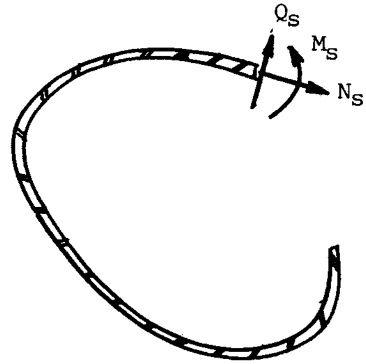

Fig. 1 Stress and moment resultants along circumferential coordinate.

difference in rigidities between a closed cross section and the same cross section with a longitudinal slit was illustrated by Tsai[4] and Reissner and Tsai.[5] The contributing factor to the rigidity change comes from contraction of the cross section. Specifically, due to anisotropy (such as differential lamina angles between layers) across the wall thickness, the shape of the cross section tends to deform owing to contraction between layers. Such a potential shape change will be counterbalanced by the presence of normal stress resultant N_s, shearing stress resultant Q_s, and moment resultant M_s, as shown in Fig. 1. When these resultants are neglected, as commonly assumed in classic beam theory, the cross section becomes free to deform. Accordingly, the rigidities between the cases with and without neglected N_s, Q_s, and M_s are significantly different. For the particular class of laminated thin-walled beam illustrated in Refs. 4 and 5, the rigidity of a closed cross section is about twice that of the same cross section with a longitudinal slit for an equivalent Poisson's ratio of 0.5. This reveals that 1) the commonly applied assumption of neglecting N_s, Q_s, and M_s for beams with isotropic material may not be applicable to beams with anisotropic material, and 2) the use of classic-beam assumptions may not be able to determine the correct beam rigidity; thin cylindrical shell theory is suggested for use in the computation of thin-walled beam rigidities if the material is anisotropic.

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